

# Active Power Loss Minimization in Radial Distribution Networks with Analytical Method of Simultaneous Optimal DG sizing

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**Abstract**—A method of reducing active power losses in distribution networks by simultaneous placement of optimally sized DGs is presented in this paper. Optimal DG sizes are calculated using analytical expressions in Python environment. IEEE 37 node test system, developed in DigSILENT PowerFactory, is used for the simulations. Three different use cases are presented in this work for validation and comparison. The active power loss reduction is compared with the base case (without any DGs). Different DG types, based on active and reactive power output, are considered during this work. The analytical expressions for simultaneous sizing proved their usefulness for loss minimization in radial networks. The complete study is done for the cases of three and four DGs placement in the system. To highlight the expanded capabilities of proposed method, it is applied to both the static and time varying load conditions.

**Index Terms**—active power loss, analytical expressions, simultaneous optimal sizing, distributed generation, radial distribution system, static load, time varying load

## I. INTRODUCTION

The continuously increasing number of Distributed Generation (DGs) in modern power system make them vital in many aspects. Along with their environmental impact, DGs influence techno-economical performance of future smart grid. As reliable and economical distribution of energy to the customers is highly desirable, it stands as the main objective of every existing and future power system. While such objectives are highly desired, penetration of DGs make it hard for the network operators to keep the operation of the system smoother. This is mainly because DGs penetration make the conventional passive distribution system as active, leading to bidirectional power flows and grid congestion. Such scenario necessitates the need of improved methods for penetration of DGs in the existing power systems.

Over the last two decades, there has been a dense research for investigating the technical aspects of power system with DGs including voltage profile improvement, active power loss minimization, power quality issues, reliability, protection and stability [1]. Most of these researches conclude that above mentioned benefit(s) can be acquired by optimizing the size

and location of the DGs in the network. Role of DGs can also be vital in providing the ancillary services such as reactive power support, frequency control and congestion management [2]. Non-optimal placement of DGs, either with respect to their size or location in the network, can cause significant rise in system power losses, voltage rise issues and other such problem [3]. Continuing the practice with non-optimal DG placement may also lead to grid reinforcement earlier than their usual time [4]. On the contrary, a significant deferring in grid reinforcement is possible by optimizing the size and location of DGs in network.

Optimizing the DG placement in network is a complex combinatorial constrained optimization problem [5]. Researchers have used different methods to solve this problem, with range of objectives. The methods range from heuristics to meta-heuristics and exact methods. For instance, authors in [6] have used cuckoo search algorithm (CSA) to find the optimal DG sizes for power loss minimization and voltage profile improvement (VPI). CSA is comparatively a new meta-heuristic method. The authors see the advantage of improved results which are achievable with fewer control parameters. Similarly, another meta-heuristics based approach is presented in [7], where authors used Fireworks Algorithm for power loss minimization and VPI. This work also focuses the need of optimal sizing and sitting of DGs.

Some researches used the optimization methods based on evolutionary and nature inspired optimization algorithms for optimal DGs placement. For example, modified plant growth algorithm based reconfiguration scheme is presented in [8]. Most of the renewable based generations are usually varying over the time, this algorithm efficiently deal with such scenarios. Other examples of the heuristic methods applied to this problem include Genetic Algorithm (GA) [9] and [10], and Particle Swarm Optimization (PSO) [11].

Based on the nature of the problem, which comprises a continuous nature in one part (location optimization) and discrete nature in other (DG size optimization), some researches suggest to distribute this problem in two sub-parts. This will

help to solve each part with a method/algorithm which is best suitable due to specific nature of the problem. One such attempt is presented in [12] where mix of Discrete PSO and GA were used. Moreover, sometimes, it is also desired under specific circumstances and requirements that the location for DG placement are kept fixed and only the sizes are optimized, as done in [13].

Analytical approach is also getting increased interest in research in recent years. This is mostly because the analytical methods are straightforward and formulation is comparatively more accurate. Another important advantage of these methods is their time to find the solution is relatively less. In [3], authors derived the analytical expression for optimal size of a DG for active power loss minimization. The same method was extended for different types of DGs in [14] where DGs of different types were placed at single location. The same method was further extended for optimizing the sizes and locations of multiple DGs in [15]. Here, the method adopted was iterative for finding multiple locations but there had been certain level of error because calculation of size does not include the impact of any DGs those will be placed in future. Such factors are improved in [16] by deriving the analytical expressions for finding the sizes simultaneously. The method was successfully applied to meshed networks.

The work presented here is an extension of [16] where the results related to active power loss minimization by different types of DGs in radial networks is presented. IEEE 37 node radial test network is taken as base system and results are validated by comparing the active power loss without DGs and with DGs by this method.

The paper is organized as follow: The detailed steps taken to carry out this simulation are given in Section II. These include the final outcome of analytical expression for finding the optimal sizes of multiple DGs for loss minimization and a complete flowchart of the other steps. Detailed information about the simulation platform along with brief description of individual tools and their working is presented in Section III. Different use cases considered for applying the presented method are explained in Section IV along with the detailed results and discussion. Finally the conclusion is presented in Section V.

## II. METHODOLOGY

### A. Optimal DG Sizes

As already mentioned, this work investigates the capabilities of previous work in reducing the active power loss by finding optimal DG sizes simultaneously in radial distribution network, the same mathematical formulation is used. For illustration purpose, the final mathematical expressions for finding optimal sizes of 3 and 4 DGs are given. This is because 3 and 4 DGs are placed in test cases to validate the expressions. It is also important to mention that the locations are taken as input in this method due to many reasons. One of such reasons includes the fact that DG locations are no more under the control of distribution network operators under current unbundled rules. Similarly, the DGs owners/investors

can have their own constraints about optimal location selection including availability of primary energy, the piece of land and/or other such factors. Although this practice is increasing, optimal location selection can provide a useful alternative to the investment in grid reinforcement, power quality improvement and other techno-economic benefits.

The optimal DG sizes are calculated using the Eq. 1 given here.

$$X = A^{-1}B \quad (1)$$

For the case of 3 DGs, the respective matrices are:

$$A = (1 + a^2) \begin{bmatrix} \alpha_{x_1x_1} & \alpha_{x_1x_2} & \alpha_{x_1x_3} \\ \alpha_{x_2x_1} & \alpha_{x_2x_2} & \alpha_{x_2x_3} \\ \alpha_{x_3x_1} & \alpha_{x_3x_2} & \alpha_{x_3x_3} \end{bmatrix}$$

$$X = \begin{bmatrix} P_{DG_1} \\ P_{DG_2} \\ P_{DG_3} \end{bmatrix} \quad B = \begin{bmatrix} B_{x_1} \\ B_{x_2} \\ B_{x_3} \end{bmatrix}$$

Whereas for the case of 4 DGs, the Eq. 1 can be completed with following matrices.

$$A = (1 + a^2) \begin{bmatrix} \alpha_{x_1x_1} & \alpha_{x_1x_2} & \alpha_{x_1x_3} & \alpha_{x_1x_4} \\ \alpha_{x_2x_1} & \alpha_{x_2x_2} & \alpha_{x_2x_3} & \alpha_{x_2x_4} \\ \alpha_{x_3x_1} & \alpha_{x_3x_2} & \alpha_{x_3x_3} & \alpha_{x_3x_4} \\ \alpha_{x_4x_1} & \alpha_{x_4x_2} & \alpha_{x_4x_3} & \alpha_{x_4x_4} \end{bmatrix}$$

$$X = \begin{bmatrix} P_{DG_1} \\ P_{DG_2} \\ P_{DG_3} \\ P_{DG_4} \end{bmatrix} \quad B = \begin{bmatrix} B_{x_1} \\ B_{x_2} \\ B_{x_3} \\ B_{x_4} \end{bmatrix}$$

Where,

$$B_{x_n} = \sum_{m=1}^{NDG} [P_{D_{x_n}} (\alpha_{x_n x_m} + \beta_{x_n x_m} a) + Q_{D_{x_n}} (\alpha_{x_n x_m} a + \beta_{x_n x_m})] - \sum_{j=1, j \neq x_n}^{Nbus} [(\alpha_{x_n j} P_j - \beta_{x_n j} Q_j)] + a \sum_{j=1, j \neq x_n}^{Nbus} [(\alpha_{x_n j} Q_j + \beta_{x_n j} P_j)]$$

$$\alpha_{ij} = \frac{R_{ij}}{|V_i||V_j|} \cos(\delta_i - \delta_j)$$

$$\beta_{ij} = \frac{R_{ij}}{|V_i||V_j|} \sin(\delta_i - \delta_j)$$

In this mathematical formulation;

- $P_i, P_j, Q_i$  and  $Q_j$  are the active and reactive power injection at  $i_{th}$  and  $j_{th}$  buses respectively
- $P_{D_{x_n}}$  and  $Q_{D_{x_n}}$  are the active and reactive power demands at  $x_n$ th bus
- $x_n$  is the set of bus where DG needs to be placed
- $V_i \angle \delta_i$  and  $V_j \angle \delta_j$  are complex voltages at respective buses
- $R_{ij}$  is the  $ij_{th}$  element of bus impedance matrix
- $NDG$  is the total number of DGs to be placed
- $Nbus$  is total number of buses in the system

Eq. 1 gives the optimal active power at the input buses, which can be used to calculate the optimal reactive power using Eq. 2.

$$Q_{DG_i} = aP_{DG_i} \quad (2)$$

Where,

$$a = (\text{sign})\tan[\cos^{-1}(pf_{DG})]$$

sign = +1 for DG injecting reactive power and -1 for DG consuming reactive power.

### B. Algorithm for Optimal Placement

A flowchart of the algorithm implemented in this paper is presented in Fig. 1. These steps find the optimal placement and losses for the static load. For time varying loads, the same method can be applied at each time step if the DG sizes need to be calculated for every load condition. Such scenario can be useful for dispatchable DGs. The locations for DG placement

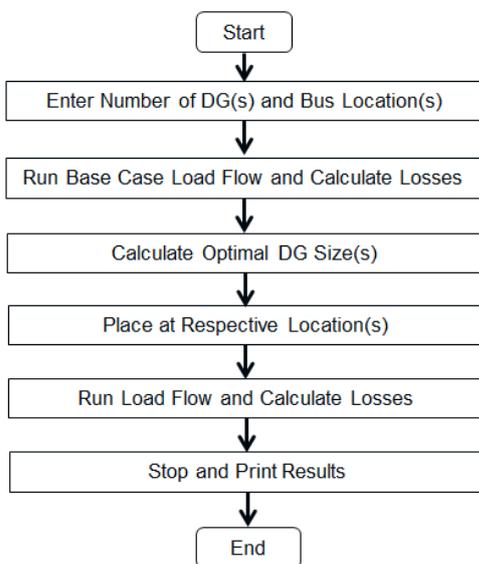


Fig. 1. Flowchart of optimal size calculation

are taken as user input due to the reasons mentioned earlier.

## III. SIMULATION SET UP

This section includes the brief description of the simulation platform and the test network. There are many software tools for simulating power networks but the selection of a tool for specific purpose needs thorough and detailed analysis [17].

### A. Power System Simulator

For simulation of power system, various tools are available. DigSILENT PowerFactory is chosen in this work because it is a dedicated tool with many functionalities that aid the simulation work. This tool is capable of efficient data management and generating various network models. Results and reporting is also a distinguishing feature with respect to ease of use. Its capability of modelling, simulating and analysing the complex power networks with different load and generator types make it a primary choice. Moreover, the developed

models and networks can be exported in different formats. Similarly, importing the models and networks of different formats is also possible with PowerFactory [18].

### B. Scripting Tool for Computing DG Sizes

PowerFactory has its own scripting language, and possibility to import models developed in Matlab or other tools. Although the scripting tool of PowerFactory has various functionalities, but, because of obvious reasons, these cannot be as much as the dedicated scripting tool can have. Based on such reasons, the researchers have tried to couple the PowerFactory with different scripting tools such as C++, Matlab [19], and Python. Some more among these coupling possibilities and their details can be found in [20].

The authors in this work preferred to use Python API implementation of the algorithm proposed in Fig. 1. The PowerFactory is initiated as an object from Python and the respective power system test case is run at desired instances as mentioned in algorithm. The calculation of optimal DG sizes and their placement is also controlled through Python. Moreover, the required data is exchanged between these tools at different instances during the program run. Finally the results are saved and used for further analysis.

### C. Power System Test Network

The proposed method is already tested over IEEE meshed networks for its ability in reducing network losses. This work utilizes the IEEE 37 node radial test system for studying the capabilities of proposed analytical expression. The network given in [21] is used with minor modifications including addition of DGs at different buses. To check the robustness and consistency of good producing results, a variety of use cases have been considered. In one of such scenarios, the test system is divided in regions, as given in Fig. 2. It is worth mentioning that this distribution is neither compulsory nor suggested. In fact, this can be considered as an option when considering DG placement due to the electrical size and/or geographically expanded size of the network. This can help in avoiding the congestion of DGs in single region (close geographical or network proximity). Moreover, sizes and shapes of region can vary in accordance with particular network, region or operator.

It is obvious from the network that the buses on main lines, originating from HV/MV transformer, are not considered in any region. This is because these lines are already heavily loaded and adding DGs there might cause line overloading. On the contrary, adding DGs in all or any number of shown regions may help in reducing their line flows. Finally, bus 37 is considered in the same region of buses from 25 - 36. In this study, the locations for DG placement are input to the algorithm.

## IV. RESULTS AND DISCUSSION

This section contains the details of the results obtained from different use cases, each with different number of DGs. DGs of different types are considered in this study to validate

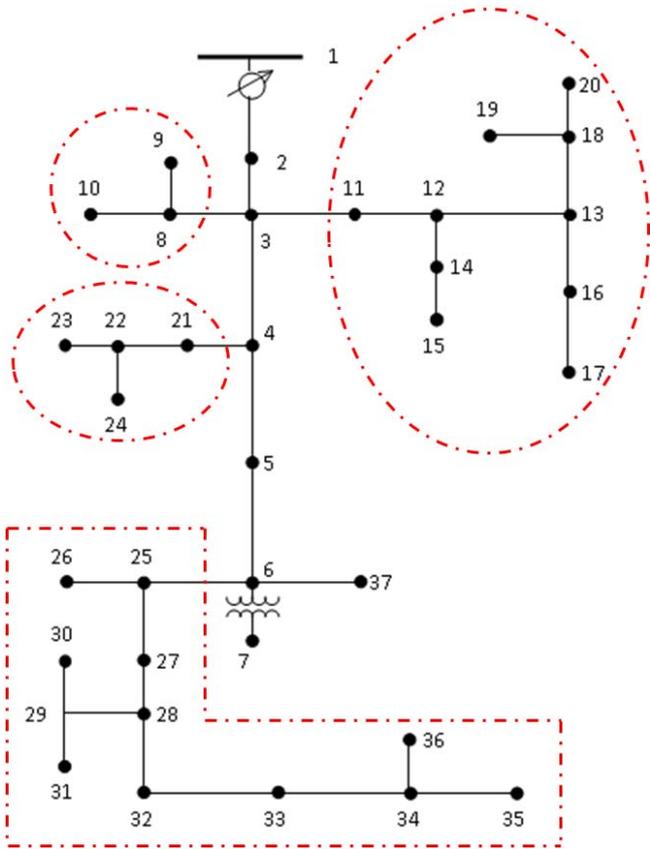


Fig. 2. Region-wise Distribution of IEEE 37 Node Radial System

the versatility of the proposed method in practical situations. Following DG types are considered:

- i. DG with active power output only (Power Factor = 1)
- ii. DG with both active and reactive power output (Power Factor = 0.95 Lead)
- iii. DG with active power output and reactive power consumption capability (Power Factor = 0.95 Lag)
- iv. DG with reactive power output only

Although the DG outputs can be taken at any power factors but due to grid code guidelines in most parts of Europe, 0.95 is taken in this work.

#### A. Use Cases

Three different uses cases considered in this work include:

- a. Complete network is considered as single unit and the locations were selected with no care of how many DGs a single feeder contains. DG sizes in this use case are calculated with the analytical expressions presented in Section II-A.
- b. In this use case, the network is considered distributed in the regions as mentioned in Section III-C and the sizes of DGs are calculated based on the analytical expressions given in Section II-A.
- c. This use case is same as use case 'b' except that the sizes of DGs are taken equal to the cumulative load in

the respective region. This is done for the comparison and validation as suggested in [13].

In these use cases, the DGs locations are taken as given in next section.

#### B. Sets of Simulation

Two different sets of simulations considered in this work.

- a. With three DGs in the network at following locations:
  - For use case a: 18, 28, 32
  - For use case b and c: 18, 22, 32
- b. With four DGs in the network at following locations:
  - For use case a: 12, 18, 22, 32
  - For use case b and c: 8, 18, 22, 32

In case of set 'a' of simulations, no DG is considered in top-left region shown in Fig. 2. This is not because of any specific reason except that the region has the least number of buses.

#### C. Simulation Results

As previously mentioned that the proposed method had been tested on same network with both the static and time varying loads. In this section, the results for each case are presented.

1) *Static Load:* The summary of optimal DG sizes found in different use cases for three and four DG locations are presented in Table I and II. During the simulations, significant reduction in active power loss is observed for most of the presented DG sizes. Fig. 3 and 4 summarize the % active power loss reduction for different DG types. This % loss reduction is calculated with respect to the base case (case without DGs) when the losses are 281.85 kW. It can be seen from the graphs that the highest reduction in losses is observed when DG with power factor of 0.95 (leading) was placed. The second best value is for the case of DG with unity power factor followed by the loss reduction for DG with lagging power factor of 0.95. This pattern remained consistent in all use cases. Moreover, the highest loss reduction is observed in use case 'a', for all types of DGs .

Another important result here is that the loss reduction is considerably low when the active power output of DG is kept to zero. This is because the method of optimal sizing finds the reactive power output with respect to the active power. Therefore, if the active power output is kept at zero, the method becomes inefficient due to non-optimal sizes.

As a final remark, the authors here would like to stress that the DGs of sizes equal to the cumulative loads in their respective regions could improve the losses significantly but not beyond the value achieved with optimal DG sizes. Only slightly better results (for three DGs only) are observed with DGs of 0.95 lagging power factor and without active power output, which are not the scenario under most of the circumstances.

2) *Time Varying Load:* The same procedure for optimal sizes has been applied to the network with time varying loads. A test load profile for usual domestic load with peak value more than 20% of that of static load is considered in this work. The time stamp started from 00:00 O'clock, taken as tic 1, and ended at 23:30 O'clock, taken as tic 48, i.e., simulations were

TABLE I  
OPTIMAL DG SIZES WITH STATIC LOAD WITH 3 DGs

Use Case a			Use Case b			Use Case c	
DG Loc	PDG (kW)	QDG (kVar)	DG Loc	PDG (kW)	QDG (kVar)	PDG (kW)	QDG (kVar)
18	0.94	0.30	18	1.46	0.47	1.81	0.34
28	0.65	0.21	22	0.96	0.31	0.93	0.19
32	0.47	0.15	32	1.33	0.42	1.27	0.35

TABLE II  
OPTIMAL DG SIZES WITH STATIC LOAD WITH 4 DGs

Use Case a			Use Case b			Use Case c	
DG Loc	PDG (kW)	QDG (kVar)	DG Loc	PDG (kW)	QDG (kVar)	PDG (kW)	QDG (kVar)
12	1.73	0.55	8	0.91	0.29	0.17	0.08
18	0.77	0.25	18	1.27	0.41	1.81	0.34
22	0.86	0.27	22	0.87	0.28	0.93	0.19
32	1.22	0.39	32	1.22	0.39	1.27	0.35

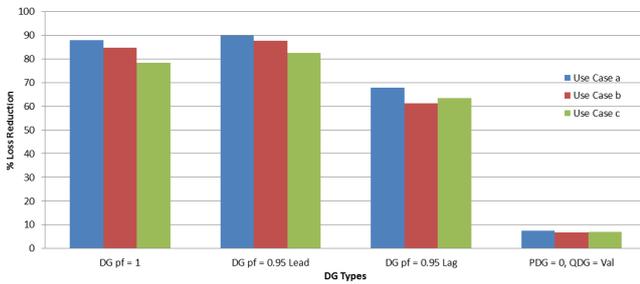


Fig. 3. Active Power Loss Percentage with Static Load with 3 DGs

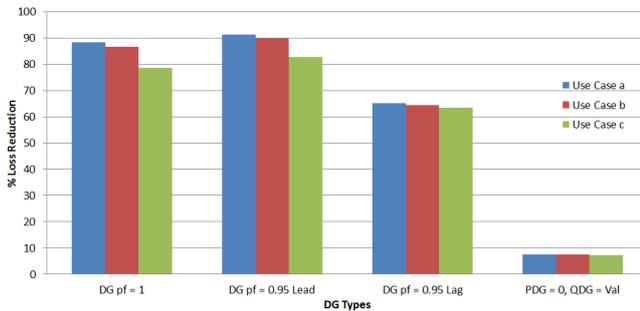


Fig. 4. Active Power Loss Percentage with Static Load with 4 DGs

carried out at half hour intervals. This is usually a fair time step in primary distribution system, even with the renewable based DGs.

Two different approaches in simulation of time varying loads are considered. In first approach, the optimal DG sizes are calculated at every time step based on the load at that instance. This yielded an optimal generation pattern for the DGs. Such an approach can be useful with dispatchable generators. In use cases 'a' and 'b', this approach is considered and results for maximum and minimum active power losses over the range of all time steps are presented in Table III and IV. Second approach is the one with DGs of constant

output, calculated for a single load value. The DG output is kept equal to cumulative load in a region for all 48 tic of load profile (same values as given in Table I and II under use case 'c' head). In this work, the results for this approach are taken in use case 'c'.

TABLE III  
ACTIVE POWER LOSS (kW) WITH TIME VARYING LOAD - 3 DGs

	No DG	DG pf = 1	DG pf = 0.95 Lead	DG pf = 0.95 Lag	PDG = 0 QDG = Val
Use Case a					
Min	75.16	34.32	28.68	56.89	68.70
Max	749.04	384.00	363.53	520.92	710.10
Use Case b					
Min	75.16	25.46	19.22	53.01	67.66
Max	749.04	81.24	70.94	212.17	688.03
Use Case c					
Min	75.16	61.18	49.14	102.66	68.47
Max	749.04	135.06	113.41	189.69	698.94

TABLE IV  
ACTIVE POWER LOSS (kW) WITH TIME VARYING LOAD - 4 DGs

	No DG	DG pf = 1	DG pf = 0.95 Lead	DG pf = 0.95 Lag	PDG = 0 QDG = Val
Use Case a					
Min	75.16	34.72	30.51	51.79	70.18
Max	749.04	218.73	204.04	315.53	698.20
Use Case b					
Min	75.16	21.16	14.67	48.19	67.07
Max	749.04	77.86	65.10	201.03	682.33
Use Case c					
Min	75.16	60.51	48.15	103.35	68.37
Max	749.04	131.80	109.85	188.01	697.22

The % active power loss reduction, calculated with respect to the active power losses without any DG are compared for all use cases of three and four DGs in Fig. 5 and 6. These graphs consider the percentages with respect to the maximum values given in Table III and IV. For DGs with unity or 0.95 leading power factor, the system produced better results with either three or four DGs. Similar response is reported for DGs without any active power output but the total loss reduction is significantly low (less than 10% of base case) as compared to other three DG types. It is also an important outcome of this study that the loss reduction is low in use case 'a' where the whole network is considered as single region. Ultimately, this highlights an additional (to those given before) reason of considering regions for DG placement. Finally, the % loss reduction is better with four DGs in comparison to three DGs case. Therefore, the optimal number of DGs is also an important point to consider the optimal placement of DGs.

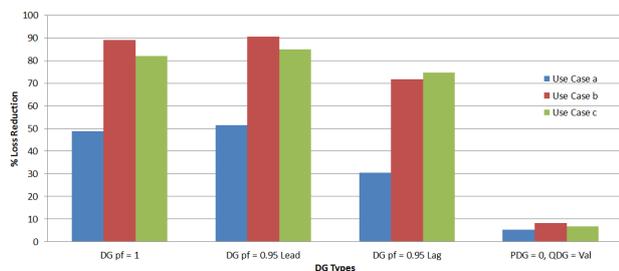


Fig. 5. Active Power Loss Percentage with Static Load with 3 DGs

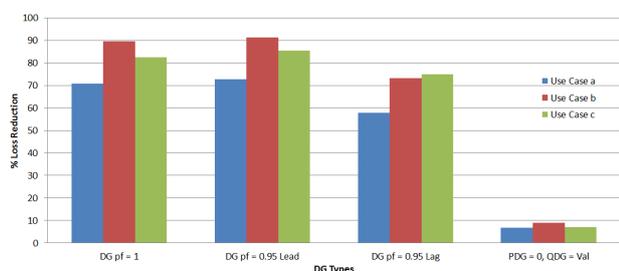


Fig. 6. Active Power Loss Percentage with Static Load with 4 DGs

## V. CONCLUSION

This paper discusses the reduction in active power loss by placing optimally sized DGs in IEEE 37 node radial distribution network. Python and PowerFactory are coupled for simulating the proposed method. The region-wise placement of DGs (use case ‘b’) produced better loss reduction in comparison to other use cases when time varying loads are considered. For the case of static loads, use case ‘a’ proved better but the difference from use case ‘b’ is not significant. Moreover, the power factor of DGs also affected the loss reductions in each of the studied cases. Best results are reported for DGs with power factor of 0.95 leading with the case of unity power factor as a closest competitor. Although results for DGs with no active power output were better than the case of no DGs, but the loss reduction was less than 10%. The % loss reduction is reported slightly better in case of 4 DGs than the case of 3 DGs, in either static or time varying load.

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